



Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

ETH RISK CENTER

LECTURE NOTES

**Fundamentals of Probabilistic Seismic Hazard
Analysis (PSHA)
Chapter 2: Lecture 5**

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Written by simona esposito
marco broccardo
IBK, ETHZ

PART III
Principles of Disaggregation and Ground Motions
Selection

4 Introduction

In the Section we outline two key elements for selecting ground motions for time history dynamic analysis of a given structure. In fact, one pillar for dynamic analysis is the input selection, which is, oftentimes, a recorded ground motion. Selection of the proper set of ground motions for a given region and level of hazard is still an open research field and source of debate. In this section, we first outline the definition of the Uniform Hazard Spectrum (UHS), followed by the concept of disaggregation, and lastly, we introduce the most used practice for ground motions selection. These concepts are central steps for correctly representing the hazard of a given region.

5 Presentation of PSHA results

Results of a PSHA analysis can be presented in different ways. The basic output of a PSHA analysis for an individual site is the annual rate of exceedance of a particular IM expressed either as rate of exceedance or return period.

As an example, Figure 18 shows the annual frequency of exceedance of PGA (unit of g), $\lambda(pga)$, for three cities in Italy (Campobasso, Napoli, and Bari) provided by the Istituto Nazionale di Geofisica e Vulcanologia (INGV). In particular, the figure shows the distribution of the 50th percentile (median map, which is the reference map for every probability of exceedance) and the distribution of the 16th and 84th percentiles. These curves are the result of the use of the Logic Tree outlined in Figure 17 for the quantification of the epistemic uncertainties. A seismic hazard map shows the variation in seismic

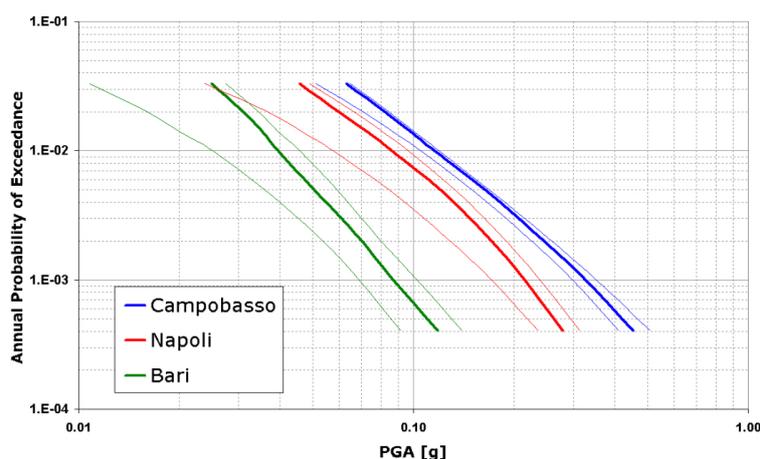


Figure 18: Seismic hazard curves for three sites in Italy. Tick lines represent median values, thin lines represent the 16th and 84th percentile of the epistemic uncertainties. (Source: <http://esse1.mi.ingv.it/data/D2.pdf>, Meletti and Montaldo, 2007).

hazard over a particular region or country. A hazard map is produced by performing hazard analysis at a large number of sites within the region under study (the task of LAB 1). A hazard curve is evaluated for each site and then the IM level determined at each point for the desired probability of exceedance and for a specific time interval is used to produce the map. Hazard contours are then drawn through the resulting values at the nodes to obtain IM level curve. Seismic hazard maps are available in literature for most areas of the world. They are usually updated as new earthquakes occur and new data are available in order to reflect the improved knowledge. As an example, Figure 19 shows the 475-year hazard map in terms of PGA (unit of [g]) for the Italian territory provided by INGV.

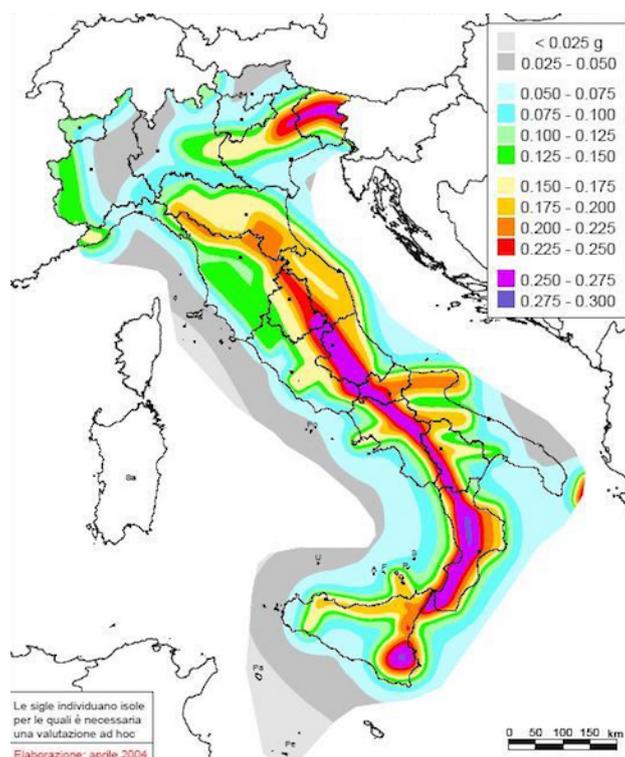


Figure 19: Seismic hazard map of the Italian national territory in terms of PGA (unit of g) considering a return period of 475 years (source: <http://esse1.mi.ingv.it>).

6 Uniform Hazard Spectrum

Response spectrum represents is one of the most widely used representation of seismic actions in earthquake-resistant design and in response analysis of buildings. There are two different ways to obtain a response spectrum.

The simplest approach, is to use the output of a PSHA analysis in terms of PGA to anchor a standard spectral shape (usually the spectrum defined by the code) at zero period, selected according to a specific soil type. The limitation of this approach is that it does not account for the variation of the shape of the spectrum with earthquake's features such as magnitude and distance. As consequence the return period of the ordinates of the spectrum is known only at a period of zero (i.e. for the PGA) while at other response periods the return period will be longer or shorter. Further, the shape of the spectrum does not change with the hazard level but only with the soil type. Eurocode 8 (CEN, 2003) tried to overcome this problem by using two spectral shapes, one for regions affected by relatively low magnitude earthquakes, the other for areas with larger earthquakes.

The Uniform Hazard Spectrum (UHS) instead, is characterized by a uniform hazard level for all the ordinates of the spectrum. The UHS is developed by first performing the PSHA many times using period-dependent GMPEs for response spectral ordinates. Then, for each period the spectral amplitude corresponding to a target rate of exceedance is identified. Those spectral values are then plotted versus the structural periods. In this manner the spectrum represents the same level of hazard across the entire range of periods. The process of constructing a UHS is illustrated by Figure 20. Figure 21 shows, instead, the UHS of the same site considered in Figure 20 for different probabilities of exceedance in 50 years (source, INGV). This spectrum is called a uniform hazard spectrum because every ordinate is characterized by an equal rate of being exceeded. However it is apparent, from their computational procedures, that UHS accounts for the occurrence of many different seismic events and then each value of the spectrum may have come from a different event. Then, it is clear that UHS cannot have a shape similar to any actual recorded signal. If, for example, the seismicity of a site is characterized by nearby sources of moderate

magnitude earthquakes and distant sources with larger events, it is possible that different portions of the UHS will actually correspond to different type of earthquakes. This characteristic has important implications for the cases in which it is necessary to represent the seismic actions in terms of acceleration time-histories.

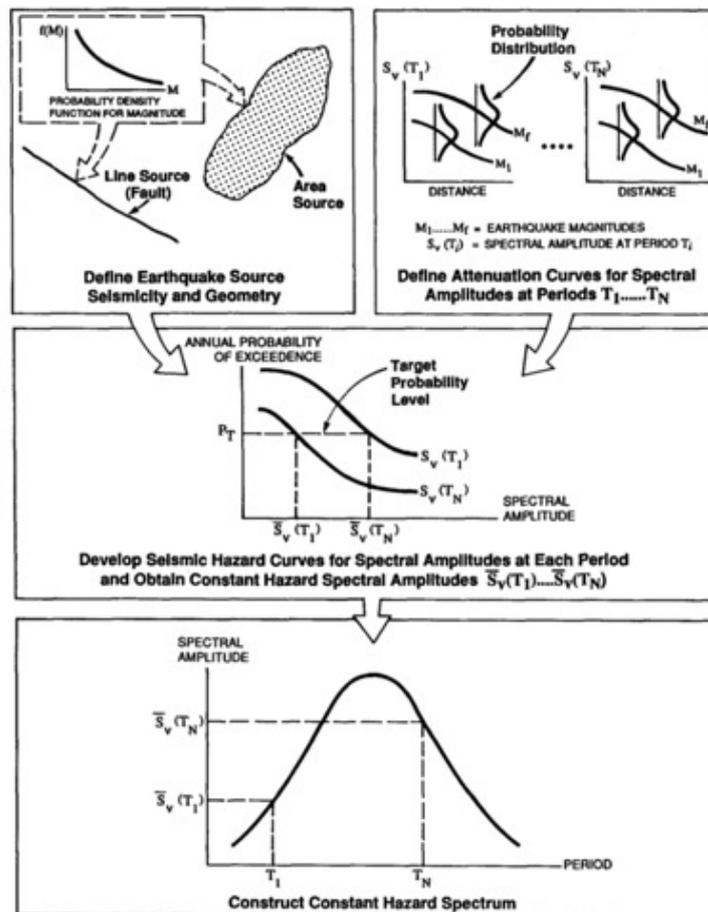


Figure 20: Illustration of the process of constructing a UHS as the output from a PSHA (EERI, 1989).

7 Disaggregation

Seismic hazard analysis leads to the hazard curve expressed in Eq. (32), (33) (Lecture Note 4) and shown in Figure 18. In some cases, it can be necessary to know the relative contributions to the hazard from different values of the random components of the problem, i.e. the magnitude, M , the source-to-site distance, R , and sometimes ϵ , the measure of the deviation of the ground motion from the value predicted by the GMPE (Lecture Note 4, Eq. (8)).

These quantities may be used, for example, to select ground motion records for response analysis. In this cases, the required analytical instrument is called *disaggregation* (or deaggregation) of seismic hazard (Bazzurro and Cornell, 1999). Disaggregation is a procedure which allows identification of the hazard contribution of each $\{M, R, \epsilon\}$ vector conditional to the exceedance of the hazard itself. The analytical result of disaggregation is the joint probability density function of $\{M, R, \epsilon\}$ conditional to the exceedance

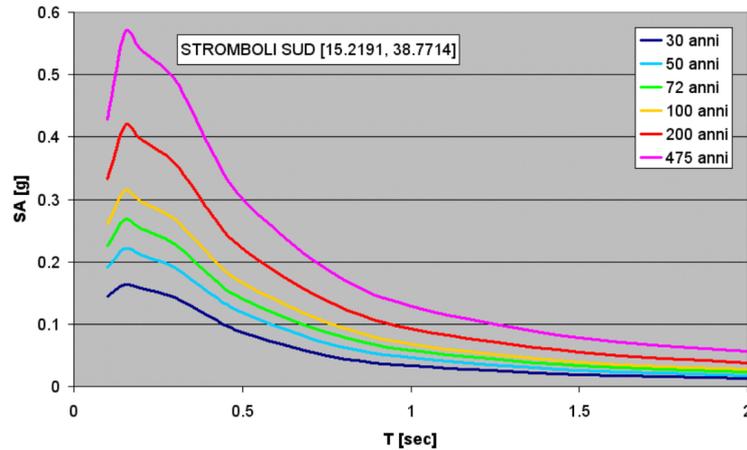


Figure 21: UHS for a site in Italy for different probabilities of exceedance in 50 years (source: <http://esse1.mi.ingv.it>, Montaldo and Meletti, 2007).

of an IM threshold (im^*), as expressed in equation (43),

$$f_{M,R,\epsilon}(m,r,\epsilon|IM > im^*) = \frac{\sum_{n=1}^{N_s} \lambda_{min}^{(n)} P[IM > im^* | m, r, \epsilon] f_{M,R,\epsilon}(m, r, \epsilon)}{\lambda(im^*)}, \quad (43)$$

where N_s is the number of seismic sources relevant for the hazard at the site, $f_{M,R,\epsilon}(m, r, \epsilon)$ is the joint PDF of $\{M, R, \epsilon\}$ and $\lambda(im^*)$ is the hazard for im^* . Practically, the disaggregation of the hazard from all N_s sources is obtained by accumulating in each $\{M, R, \epsilon\}$ bin the contribution to the global hazard, during the numerical integration of Eq. 32, (i.e. equations 34-35, Lecture Note 5).

Equation (43) computes the relative contribution to the hazard originated by a specific characterization of the seismicity. When epistemic uncertainty is of concern, in principle, it is possible to disaggregate the hazard from each considered seismicity model (i.e. for each brunch of a Logic Tree). In realistic cases, this could be impractical, due to the very large number of cases considered. Then, the hazard values usually considered correspond to the 50th percentile of the hazard distribution obtained by using a specific logic tree. Thus, the disaggregation is performed using the inputs along the logic tree path that provided hazard values closest to the reference 50th percentile hazard.

As an example, Figure 22 shows the disaggregation of median PGA for a site in Italy (Spallarossa and Barani, 2007), in terms of magnitude and distance, with a probability of exceedance of 10% in 50 years, provided by the Istituto Nazionale di Geofisica e Vulcanologia (INGV). Note that in this case, the disaggregation has been applied using the inputs along the logic tree path that provided hazard values closest to the reference 50th percentile hazard.

7.1 Magnitude disaggregation: an example

In case of magnitude disaggregation, we might be interested to calculate the probability of having an earthquake magnitude equal to m , given that the ground motion $IM > im^*$ has occurred, i.e.:

$$\begin{aligned} P(M = m | IM > im^*) &= \\ &= \frac{\lambda(M > im^* | M = m) P(M = m)}{\lambda(im^*)} \\ &= \frac{\sum_{n=1}^{N_s} \sum_{m=1}^{N_r} \lambda_{min}^{(n)} P[IM > im^* | M^{(n)} = m^{(n)}, R^{(n)} = r_m^{(n)}] P(M^{(n)} = m^{(n)}) P(R^{(n)} = r^{(n)})}{\lambda(im^*)} \end{aligned} \quad (44)$$

This is practically equal to the rate of earthquakes with $IM > im^*$ and $M = m$, divided by the rate of all earthquakes with $IM > im^*$, i.e. $\lambda(im^*)$.

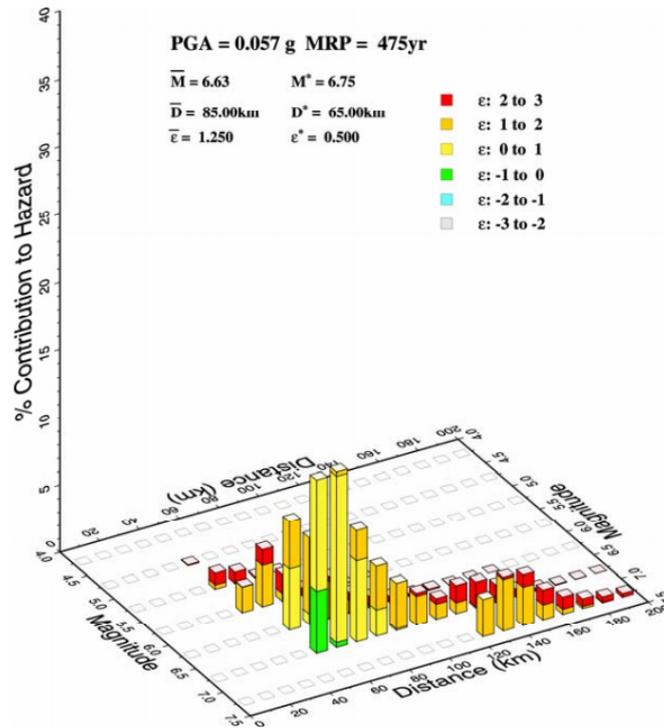


Figure 22: Disaggregation of median PGA with probability of exceedance of 10% in 50 years (source: <http://esse1.mi.ingv.it/>, Spallarossa and Barani, 2007).

Example

(Adapted from Baker). Compute the relative contributions of two linear faults to $\lambda(PGA) > 1[g]$. Fault 1 produces earthquakes with magnitude $M = 6.5$ and source to site distance (closest distance) $r_1 = 10[\text{km}]$ from the site; and has an annual occurrence rate of 0.01; Fault 2 produces earthquakes with magnitude $M = 7.5$ and source to site distance (closest distance) $r_2 = 20[\text{km}]$ from the site, and it has annual occurrence rate of 0.002.. Events will rupture all the fault (the distance is a constant). The used GMPE is (Cornell et al., 1979):

$$\begin{aligned} \overline{\ln PGA} &= -0.152 + 0.859M - 1.803 \ln(R + 25) \\ \sigma &= 0.57 \end{aligned} \quad (45)$$

From Eq. (33) (hazard integral) considering $\lambda_{min}^{(1)} = 0.01$ and $\lambda_{min}^{(2)} = 0.002$, we obtain

$$\lambda(PGA > 1[g]) = 0.01P[PGA > 1[g]|6.5, 10] + 0.002P[PGA > 1[g]|7.5, 20], \quad (46)$$

where

$$P[PGA > 1[g]|6.5, 10] = 1 - \Phi\left(\frac{\ln 1 - \ln(0.3758)}{0.57}\right) = 1 - \Phi(1.72) = 0.043 \quad (47)$$

$$P[PGA > 1[g]|7.5, 20] = 1 - \Phi\left(\frac{\ln 1 - \ln(0.5639)}{0.57}\right) = 1 - \Phi(1.01) = 0.158. \quad (48)$$

Finally $\lambda(PGA > 1[g]) = 0.01(0.043) + 0.002(0.158) = 0.000746$

Magnitude Disaggregation

The relative contributions of the two linear faults to exceedance of $PGA = 1[g]$ can be computed as following:

$$P[M = 6.5 | PGA > 1[g]] = \frac{0.01P[PGA > 1[g]|6.5, 10]}{\lambda(PGA > 1[g])} = 0.58 \quad (49)$$

$$P[M = 7.5 | PGA > 1[g]] = \frac{0.002P[PGA > 1[g]|7.5, 20]}{\lambda(PGA > 1[g])} = 0.42. \quad (50)$$

Then the more active but smaller source (Fault 1) makes a greater contribution to exceedance of the $PGA = 1[g]$.

7.2 Design earthquakes and engineering applications

Historically, mean values of M , R , and ε of the disaggregation, $\{\bar{M}, \bar{R}, \bar{\varepsilon}\}$ has been the most popular contender for the role of defining the dominant event, i.e. the design earthquake. Today modal values are preferred because respect to the mean values, they corresponds to a realistic scenario. A Design Earthquake (DE), defined as mode of the disaggregation joint PDF (equation (43)), is the vector $\{M^*, R^*, \varepsilon^*\}$ that gives the largest contribution to the hazard (i.e. to the exceedance of the $IM = im$ threshold corresponding to the considered return period).

As an example, Figure 23 shows maps of DEs in terms of mean $(\bar{M}, \bar{R}, \bar{\varepsilon})$ and modal $(M^*, R^*, \varepsilon^*)$ values as result of the disaggregation of the median PGA with probability of exceedance of 10% in 50 years computed for the Italian territory (source INGV).

In some cases, the use of single statistics, such as the mean or the mode is not sufficient to describe the characteristics of the ground motions that most likely to threaten the site. For example, disaggregation of the joint PDF may show more than a single mode that significantly contribute to the hazard. In this case a second mode, i.e. a second DE is defined as the second relative maximum of $f_{M,R,\varepsilon}(m, r, \varepsilon | IM > im^*)$ distribution.

Design earthquakes represent nowadays the preliminary criterion for record selection for assessing the nonlinear demand of structures. The current state of practices is based on first disaggregating the seismic hazard at the site for the level of spectral acceleration (at the first mode period of the structure) at a specified probability (e.g. 10% in 50 years). Then records are selected to match within tolerable limits the mean or modal value of the M and R (from a disaggregation analysis) and other source, path and site characteristics such as, style of faulting, site conditions etc. Finally the selected records are scaled to match, in some average sense, a target spectrum (e.g. code spectrum or UHS).

Another possible use of design earthquakes is the possibility to build hazard curves for secondary intensity measures conditional to design value of primary intensity measure for which a hazard curve is available by national authorities. In fact, in case a secondary IM is required to improve the estimation of the structural response and hazard curves are not available for this IM , conditional hazard maps can be built computing the probabilistic distribution for the secondary IM conditional to the design value of the primary IM . An example of this application is reported in Iervolino et al., 2011.

For further details about disaggregation procedures and applications, the reader is addressed to (Bazzurro and Cornell, 1999 and Iervolino et al., 2011).

8 Record Selection for Dynamic Response Analysis

The most complete analysis for the safety level of existing civil structures requires nonlinear dynamic analysis. This type of analysis requires a detailed modeling of the structure and a proper selection and scaling of seismic input. The main steps for record selection and scaling are summarized below and shown in Figure 25 (Danciu and Fah, 2016):

1. Define the target spectrum for the site and the limit state of interest
 - Usual target spectrum are: design-code spectrum, UHS, and Conditional Spectrum (Baker, 2010).

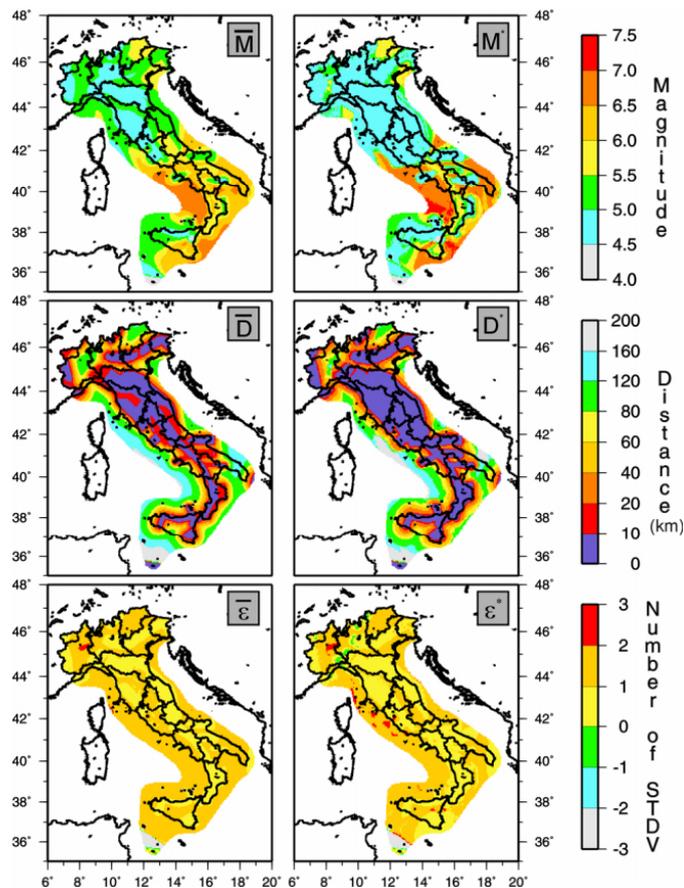


Figure 23: Mean (\bar{M} , \bar{R} , $\bar{\epsilon}$) and modal (M^* , R^* , ϵ^*) values of the disaggregation of median PGA with probability of exceedance of 10% in 50 years (source: <http://esse1.mi.ingv.it/>, Spallarossa and Barani, 2007).

2. Assessment of the dominant earthquake characteristics (e.g. magnitude, distance etc.) which dominate the hazard for the spectral ordinate of interest.
 - From seismic hazard disaggregation (if available), otherwise from experience.
3. Choose a final reduced subset of ground motions matching the target spectrum T^* (or over a range of periods)
 - Ground motion signals can be real (better), synthetic, or artificial. We suggest to use artificial earthquake (i.e., the one derived by stochastic process) to augments a given subset rather than be used in substitution of real ground motions.
4. Modify the ground motion so that its response spectrum more closely matches the design spectrum (this is a conventional but controversial practice).
 - Amplitude based, or response spectrum (frequency-domain or by time-domain) modification methods.

8.1 Selection

Generally, the signals that can be used for the seismic structural analysis are of three types: artificial waveforms; simulated accelerograms (synthetic); and real (natural) records. Many codes worldwide, such

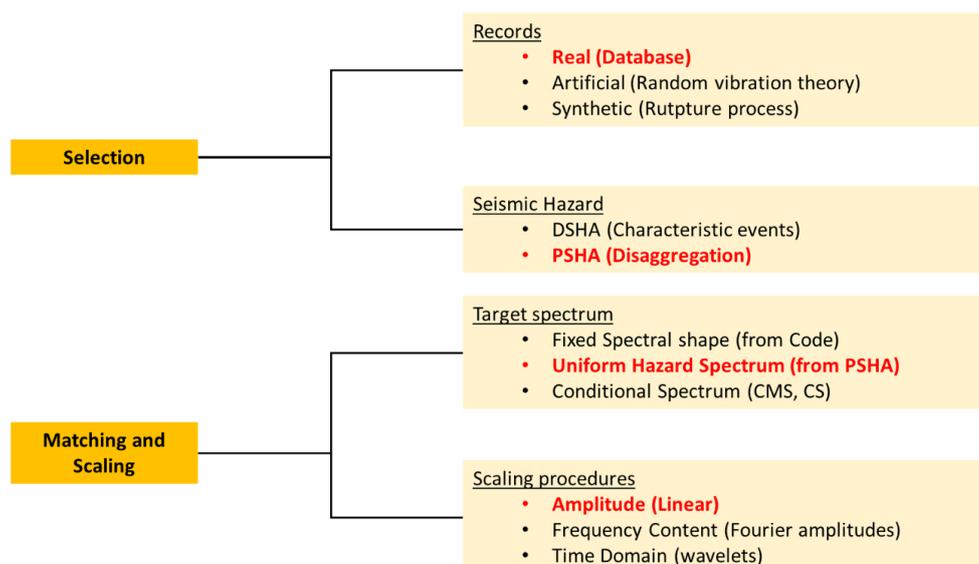


Figure 24: Overview of the main steps of record selection and matching for dynamic seismic analysis of structures. (source: Danciu and Fah, 2016).

as Eurocode 8 (EC8), allows the use of all the three types of signals listed above. Artificial signals are usually obtained generating a Power Spectral Density (PSD) from the code response spectrum, and deriving signals compatible to that. Simulated accelerograms are obtained via modelling of the seismological source and may account for path and site effects. The last type are ground-motion records from real events. There are many on-line, user friendly databases of strong-motion recording; however an issue regarding the use of these signals, whose spectra are generally non-smoothed, is the selection of a set compatible with a code-specified spectrum. The basic criteria for selecting ground motions are formulated on basis of earthquake source parameters of a design scenario defined either deterministically (through DSHA) or probabilistically (through PSHA). Design scenario is usually defined in terms of : a) source parameters, i.e. magnitude range of significant event(s), focal depth, style of faulting, directivity; b) Path: distance range of the site from the causative fault(s), fault azimuth, geometrical spreading and attenuation; c) Site: surface geology (generally described by average shear-wave velocity), topography.

Probabilistic methods require hazard disaggregation, i.e seismic hazard is first disaggregated at the site (Bazzurro and Cornell, 1999), by causative magnitude (M) and distance (R), for the level of spectral acceleration (at the first mode period of the structure, T^*) at a specified probability (e.g. 10% chance of exceedance in 50 years). The records are then chosen to match within tolerable limits the mean or modal value of the M and R and site conditions, i.e., the expected value or most likely value of these characteristics given that exceedance.

8.2 Matching

The selected set of records has then to match a predefined target response spectrum, which represents the link between the structure and the ground motion. The target spectrum can be defined by i) a design spectrum as specified by seismic design code provisions, ii) uniform hazard spectrum resulting from seismic hazard assessment and more recently by iii) a conditional mean spectrum. Design code spectrum is the simplest target spectrum and it is provided by seismic design codes and reference standards. Design spectra are generally determined by smoothing or enveloping multiple earthquake response spectra. Thus, the design spectra are typically conservative as they envelope spectral ordinates of equal probabilities of exceedance. The UHS, as introduced above, is generated by a PSHA and represents the response spectra for a specified probability of exceedance (or mean return period). During the last years, the ground motion epsilon, ε , has been used as an indicator of the spectral shape, which lead to the development

of the Conditional Mean Spectrum (CMS) as target response spectrum. The CMS is a spectrum that considers ε at a single conditioning period and the correlation of spectral ordinates at different periods to compute the conditional mean and standard deviation of the spectral ordinates at all other periods, given the conditioning period. The conditioning period is usually chosen as the fundamental (1st mode) period of the structure, although other periods could be used.

8.3 Scaling

When the number of eligible records selected is not reasonable to adequately evaluate the structural response, scaling methods are applied¹. The most common scaling procedures are amplitude-matching and spectrum-matching techniques. Spectrum matching is done either by modifying spectral ordinates in the frequency domain (FD) or adding (subtracting) wavelets in the time domain (TD). In both ways, the spectral shape is modified to match the target spectrum. The amplitude-matching procedure (Figure 8) linearly scales the amplitude of the ground motions to match the target spectrum. To this aim, a scale factor (SF) has to be determined to scale the ground motions to the design response values. The match

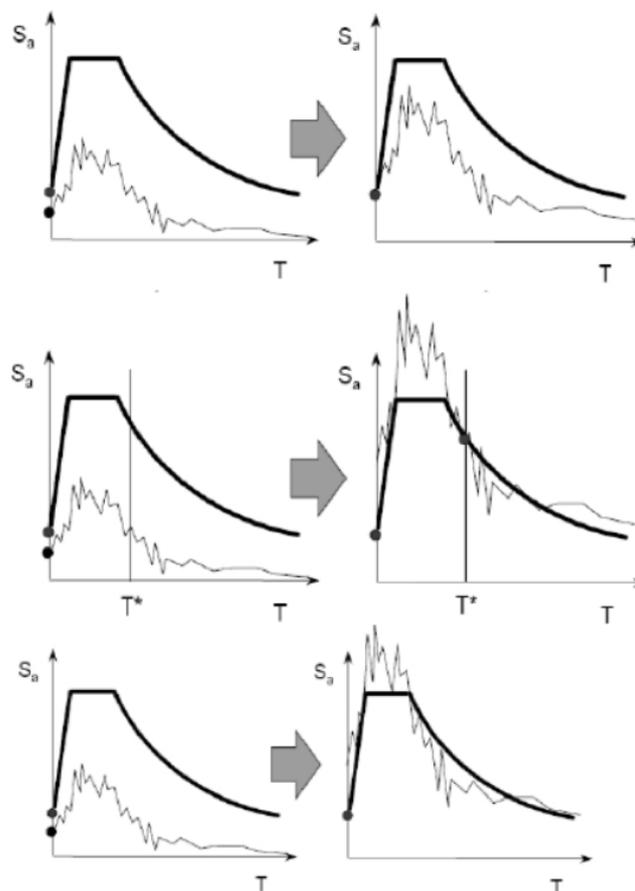


Figure 25: Amplitude-based scaling: a) based on PGA; b) based on T^* ; c) based over a range of periods $T_1 - T_2$. (Source)

can be based on (a) PGA, (b) the first fundamental period of the structure T^* or (c) minimizing the

¹Alternatively artificial ground motions can be used

misfit (least square error) over a range of periods $T_1 - T_2$, as expressed in the following equation:

$$r^2 = \int_{T_1}^{T_2} [S_a(T)_{design} - SF Aa(T)_{unscaled}]^2 dt \quad (51)$$

$$r^2 = \sum_{i=1}^N [S_a(T_i)_{design} - SF Aa(T_i)_{unscaled}]^2 \quad (52)$$

$$SF = \frac{\sum_{i=1}^N [S_a(T_i)_{design} - SF Aa(T_i)_{unscaled}]^2}{\sum_{i=1}^N [SF Aa(T_i)_{unscaled}]^2} \quad (53)$$

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